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Section A

Noise of a bolometer with vanishing self-heating

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We discuss the thermal noise of a low temperature magnetic bolometer, which has the property of not being heated by the thermometer measurement power. This bolometer is promising for measuring small energy pulses from absorbed single particles or X-rays. Having thermal noise only we can calculate the uncertainty of energy detection and we show that this is not limited by the quantity $\sqrt{4kT^2C}$. This result is of interest with respect to experiments which demand large absorbers.

1. Introduction

Bolometers with resistive thermometers have been used for a long time, and their ultimate sensitivity has been investigated in detail [1,2]. In addition to the thermal noise, the Johnson noise is important in those bolometers and determines the optimum experimental conditions. In recent years a bolometer has been developed which has practically no power input from measurement, and therefore its ultimate energy resolution is an interesting problem. The energy resolution is not limited by the usual thermodynamic limit $\sqrt{4kT^2C}$, where C is the heat capacity of the bolometer [3]. This is an important point, because the bolometer should be useful for applications demanding large absorbers. In this bolometer magnetic thermometry is applied, using a dc-SQUID of special design. The experimental method, as well as the design of the bolometer, and measurements with single particles and X-ray quanta can be found in earlier papers ([4] and references therein).

Other researchers have already investigated other types of bolometers that have small heat input from the measurement process. These are e.g. bolometers with pyroelectric thermometers [9] and superconducting meander line inductance thermometers [10]. These might share some of the properties and advantages of the magnetic bolometer reported here. But in order to find out whether this is the case a detailed consideration of the heat input and of the involved heat capacities and thermal conductances would be necessary.

In a very recent paper [3] some basic considerations on the noise and the possible energy resolution of the magnetic bolometer have already been discussed. Due to the lack of bias heating one can in principle use an arbitrarily small thermal conductance between the bolometer and the bath. On that account the spectral density of energy fluctuations has high amplitudes only at low frequencies, but it is very low at higher frequencies where the signal is measured. In this paper we discuss the noise and the resulting energy resolution. First we give in section 2 an estimation of the upper limit of the power which charges the thermometer. In section 3 we schematically show the experimental setup, and the corresponding model for the calculations. Then in section 4 we give a detailed calculation of the noise and the resulting energy resolution. Further we discuss the limitations given by the SQUID-sensor. In the final section 5 we discuss experimental problems, in particular with respect to the magnetic samples.

2. Self-heating

The bolometer self-heating has been noted above to be practically zero. There is a small effect due to the ac-current in the circuit used to measure the change in magnetic moment. These currents are induced by the room temperature electronics used to measure the dc-SQUID, and create a time dependent magnetic field. In order to give an estimate of its magnitude we consider a particular magnetic sample with a high number of magnetic atoms, $N_m = 1.9 \times 10^{20}$, and atomic moments $\mu = 1.84\ \mu_B$ (μ_B

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= Bohr magneton). These numbers correspond to a sample (250 mg of CMN) which is appropriate for a compound detector with a very large absorber as discussed in ref. [4]. With that we can calculate the power loss in an alternating field $B_1 \exp(-i\omega t)$ in parallel with the constant magnetic field, according to a formula given in ref. [5], and rewritten in the form

$$P(\omega)=\frac{1}{2}\frac{N_m(\mu B_1)^2}{3kT}f(\omega), \tag{1}$$

with

$$f(\omega)=\frac{1}{\tau_1}\frac{(\omega\tau_1)^2}{1+(\omega\tau_1)^2}. \tag{2}$$

k is Boltzmann's constant and τ_1 is the spin lattice relaxation time, which is of the order of 1 s at the temperatures of measurement. A magnetic flux white noise with a rms value of 3×10^{-6} $\Phi_0/\sqrt{\text{Hz}}$ (Φ_0 = superconducting flux quantum) has been measured at frequencies $f > 1$ Hz [4], and that corresponds to a fluctuating field with the amplitude $B_1 = 2 \times 10^{-15}$ T/$\sqrt{\text{Hz}}$.

With these data we find from Eq. (2): $P(\omega) = 6.3 \times 10^{-32} f(\omega)$ W/Hz. When summing up over frequencies we can take $\omega_g = 2\pi \times 1000$ Hz as the upper limit. Higher frequencies are cut off with filters because the fastest signals we obtain with the magnetic bolometer are of the order of 1 ms. With $\int f(\omega)\,d\omega = 2\pi \times 10^3$, the total power loss is found as $P = 4 \times 10^{-28}$ watts. A power of less than 10^{-27} W can be considered as zero, because it is much lower than the mean absorbed power from radioactive background, even in an underground laboratory.

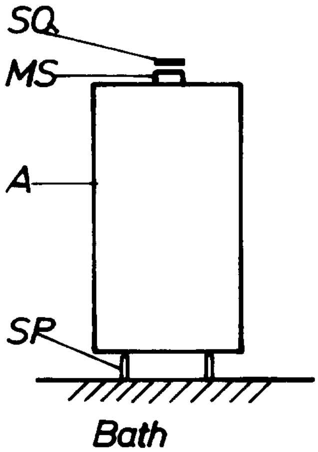

Fig. 1. Schematic design of the magnetic bolometer. A: absorber, MS: magnetic sample, SP: supporting pins, SQ: SQUID. The detector is placed in a superconducting hollow cylinder (not shown) with a frozen-in magnetic field.

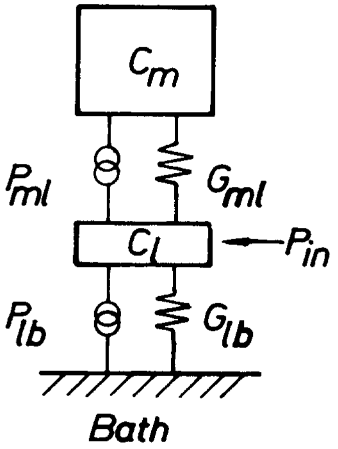

Fig. 2. The equivalent circuit of the detector with the heat capacities C_l, C_m, the thermal conductances G_{lb}, G_{ml}, and the corresponding noise generators P_{lb}, P_{ml}. The power input from absorbed particles is indicated by P_{in}.

3. The bolometer and its theoretical model

Fig. 1 shows schematically the design of the bolometer. It consists of a large absorber and a small paramagnetic sample maintained in thermal contact by a layer of glue. The detector is put on supporting pins, and the contact points represent the limited thermal conductance to the bath. In front of the magnetic sample (but with no connection to it) is the SQUID which measures the change of the magnetization on particle absorption. Notice that no wires are connected with the detector itself.

When a particle hits the absorber, most of its energy becomes thermalized within a short time, i.e. the energy quickly heats up the lattice of the compound detector. The corresponding heat capacity, denoted as C_l should therefore include the absorber, the glue and the lattice of the magnetic sample. Then on a longer time scale most of the energy is transferred to the magnetic moments (with an internal degree of freedom) which actually represent the thermometer. Its heat capacity C_m is much higher than C_l at low temperatures, because $C_m \sim T^{-2}$ whereas $C_l \sim T^3$.

Fig. 2 shows the equivalent circuit of the detector. We will use the following notations: G_{ml} = thermal conductance from C_m to the lattice; P_{ml} = noise generator for the thermal noise in G_{ml}, G_{lb} = thermal conductance between lattice and bath; P_{lb} = noise generator for G_{lb}, P_{in} = input power of thermalized particle energy; T_m = temperature of the magnetic system; T_l = temperature of the lattice; T_b = bath temperature. We assume $T_m \cong T_l \cong T_b$, because only small absorbed energies are considered, and therefore the heat capacities and the thermal conductances are constant at a fixed bath temperature; furthermore the equations of motion are linearized with this assumption. For convenience we now use the same symbols to represent the small fluctuating parts of the temperatures.

4. Noise analysis

According to the presumptions given above, the time dependent energies in both subsystems with $E_{\mathrm{m}} = C_{\mathrm{m}} T_{\mathrm{m}}$ and $E_{\mathrm{l}} = C_{\mathrm{l}} T_{\mathrm{l}}$ are described by the equations

$$C_{\mathrm{m}}\,\mathrm{d}T_{\mathrm{m}}/\mathrm{d}t = \mathrm{d}E_{\mathrm{m}}/\mathrm{d}t = \mathrm{j}\omega C_{\mathrm{m}} T_{\mathrm{m}} = (T_{\mathrm{l}} - T_{\mathrm{m}})G_{\mathrm{ml}} + P_{\mathrm{ml}}, \qquad (3)$$

$$C_{\mathrm{l}}\,\mathrm{d}T_{\mathrm{l}}/\mathrm{d}t = \mathrm{d}E_{\mathrm{l}}/\mathrm{d}t = \mathrm{j}\omega C_{\mathrm{l}} T_{\mathrm{l}} = -(T_{\mathrm{l}} - T_{\mathrm{m}})G_{\mathrm{ml}} + (T_{\mathrm{b}} - T_{\mathrm{l}})G_{\mathrm{lb}} - P_{\mathrm{ml}} + P_{\mathrm{lb}} + P_{\mathrm{in}}. \qquad (4)$$

We assume sine wave excitations and solve independently at each frequency, so $\mathrm{d}/\mathrm{d}t = \mathrm{j}\omega$ and assume $T_{\mathrm{b}} = 0$. These equations can be solved immediately by eliminating the unknown lattice temperature T_{l} and one finds

$$T_{\mathrm{m}}\left[\left(1 + \frac{\mathrm{j}\omega C_{\mathrm{m}}}{G_{\mathrm{ml}}}\right)(G_{\mathrm{lb}} + \mathrm{j}\omega C_{\mathrm{l}}) + \mathrm{j}\omega C_{\mathrm{m}}\right] = P_{\mathrm{in}} + P_{\mathrm{lb}} + P_{\mathrm{ml}}\left(\frac{\mathrm{j}\omega C_{\mathrm{l}} + G_{\mathrm{lb}}}{G_{\mathrm{ml}}}\right). \qquad (5)$$

Defining the square bracket in Eq. (5) as D, the equation can be written as

$$T_{\mathrm{m}} = \left\{P_{\mathrm{in}} + P_{\mathrm{lb}} + P_{\mathrm{ml}}\left(\frac{\mathrm{j}\omega C_{\mathrm{l}} + G_{\mathrm{lb}}}{G_{\mathrm{ml}}}\right)\right\}\frac{1}{D}. \qquad (5.a)$$

The function $1/D$ is the responsitivity in units K/W.

The powers P_{lb} and P_{ml} are random and incoherent with each other and may be expressed as spectral densities, so that *SD*(E_{m}) is the spectral density of E_{m}. We thus find in the absence of power input, $P_{\mathrm{in}} = 0$,

$$\mathrm{SD}(E_{\mathrm{m}}) = C_{\mathrm{m}}^2 \mathrm{SD}(T_{\mathrm{m}}) = C_{\mathrm{m}}^2\left[\mathrm{SD}(P_{\mathrm{lb}}) + \mathrm{SD}(P_{\mathrm{ml}})\left|\frac{(\mathrm{j}\omega C_{\mathrm{l}} + G_{\mathrm{lb}})}{G_{\mathrm{ml}}}\right\|\right]\frac{1}{|D|^2}. \qquad (6)$$

We can put the ideal phonon noise formula in unit bandwidth

$$\mathrm{SD}(P_{\mathrm{lb}}) = 4kT^2 G_{\mathrm{lb}} \qquad (7.a)$$

$$\mathrm{SD}(P_{\mathrm{ml}}) = 4kT^2 G_{\mathrm{ml}} \qquad (7.b)$$

to obtain the general result, but that will be done later.

First we want to consider the limit $C_{\mathrm{l}} \to 0$, because $C_{\mathrm{l}} \ll C_{\mathrm{m}}$ has been demonstrated in the experiments. Under this condition the lattice has the function only to thermalize the energy of an absorbed particle and to transfer it to the magnetic system. In this limit it is easily found that Eq. (6) reduces to the simpler form

$$\mathrm{SD}(E_{\mathrm{m}}) = \frac{C_{\mathrm{m}}^2\left[\mathrm{SD}(P_{\mathrm{lb}}) + \mathrm{SD}(P_{\mathrm{ml}})(G_{\mathrm{lb}}/G_{\mathrm{ml}})^2\right]}{\left[G_{\mathrm{lb}}^2 + \omega^2 C_{\mathrm{m}}^2 (1 + G_{\mathrm{lb}}/G_{\mathrm{ml}})^2\right]}. \qquad (8)$$

Introducing now the terms of Eqs. (7.a) and (7.b) for the spectral density of the phonon noise, we find

$$\mathrm{SD}(E_{\mathrm{m}}) = \frac{4kT^2 C_{\mathrm{m}}^2}{G_{\mathrm{tot}}}\,\frac{1}{1 + (\omega C_{\mathrm{m}}/G_{\mathrm{tot}})^2} = kT^2 C_{\mathrm{m}}\frac{4\tau_{\mathrm{b}}}{1 + (\omega\tau_{\mathrm{b}})^2}, \qquad (9)$$

where we have introduced $\tau_{\mathrm{b}} = C_{\mathrm{m}}/G_{\mathrm{tot}}$, with

$$G_{\mathrm{tot}} = G_{\mathrm{ml}} G_{\mathrm{lb}}/(G_{\mathrm{ml}} + G_{\mathrm{lb}}). \qquad (10)$$

In the limit $C_{\mathrm{l}} \to 0$, the total thermal conductance between C_{m} and the bath is given by G_{tot}, as can be seen immediately from Fig. 2. With that, $C_{\mathrm{m}}/G_{\mathrm{tot}} = \tau_{\mathrm{b}}$ is the time constant for the heat flow from C_{m} to the bath, which determines the decay time of the signal as found in the experiment. At the same time τ_{b} strongly influences the spectral density of the energy fluctuations as given in Eq. (9).

In addition one can prove that this spectral density gives the right thermodynamic limit. The total energy fluctuation in the magnetic system is known to be $\sqrt{4kT^2 C_{\mathrm{m}}}$. We recover this by integrating the spectral density:

$$(\Delta E_{\mathrm{m}})^2 = \int_0^\infty \mathrm{SD}(E_{\mathrm{m}})\,\mathrm{d}f = kT^2 C_{\mathrm{m}}\int_0^\infty \frac{4\tau_{\mathrm{b}}}{1 + (\omega\tau_{\mathrm{b}})^2}\,\mathrm{d}f = kT^2 C_{\mathrm{m}}. \qquad (11)$$

Having shown that our approach gives the right limits, we go on to explain that one in principle can measure the particle energy to better than $\sqrt{4kT^2 C_{\mathrm{m}}}$ resolution, as has been noticed in earlier papers [2,3]. The physical argument is that the thermodynamic fluctuations have an equivalent bandwidth $B = 1/(4\tau_{\mathrm{b}})$, while the pulses have a much larger bandwidth. Thus, if it is possible to measure E_{m} accurately enough at a large enough bandwidth, then there is no fundamental limit in the energy resolution.

We now calculate the ultimate limit of sensitivity due to thermal noise. For this we need the noise equivalent power sensitivity (or energy sensitivity) of the magnetic system, referred to the crystal lattice where particle heat is delivered. The ultimate sensitivity of the detector is then found from

$$(\Delta E_{\mathrm{m}})^{-2} = \int_0^\infty \frac{1}{\mathrm{NEP}^2(f)}\,\mathrm{d}f, \qquad (12)$$

for a pulse of unit energy. The argument that leads to this formula is that each individual frequency band of 1 Hz gives equally valid measurement of the pulse height, and the signal to noise ratio of a pulse of unit energy is $1/\mathrm{NEP}^2(f)$ in each unit bandwidth. Adding up the measurements at all frequencies gives the result. One can see that if NEP is limited only by the phonon noise, which is independent of frequency, then the integral can be very large and the energy uncertainty very small.

To find $\mathrm{NEP}^2(f)$ we refer to Eq. (5). The right hand side of Eq. (5) gives the relationship of the noise generators to the input power P_{in}. The last term on the right hand side shows the effects of random phonon exchange between the lattice and the magnetic system. If the SQUID-sensor is ideal and noise free, then the sensor gain does not matter, and only the thermodynamic noise terms P_{lb} and P_{ml} matter. These terms are added quadratically because they are uncorrelated. We use the expression for P_{lb}^2 and P_{ml}^2 per unit bandwidth, which are equivalent to $\mathrm{SD}(P_{\mathrm{lb}})$ and $\mathrm{SD}(P_{\mathrm{ml}})$, respectively, as given by Eqs. (7.a) and (7.b). Hence

$$\mathrm{NEP}^2(f) = 4kT^2\left[G_{\mathrm{lb}} + \frac{1}{G_{\mathrm{ml}}}\left|G_{\mathrm{lb}} + \mathrm{j}\omega C_{\mathrm{l}}\right|^2\right]$$

$$= 4kT^2\left[G_{\mathrm{lb}} + \frac{1}{G_{\mathrm{ml}}}\left(G_{\mathrm{lb}}^2 + \omega^2 C_{\mathrm{l}}^2\right)\right]. \qquad (13)$$

With the help of the formula $\int \mathrm{d}x/(a^2 + x^2) = (1/a)\arctan(x/a)$, Eq. (12) can be solved, and it yields:

$$(\Delta E)^2 = 4kT^2 C_{\mathrm{l}} 4\sqrt{G_{\mathrm{lb}}/G_{\mathrm{ml}} + (G_{\mathrm{lb}}/G_{\mathrm{ml}})^2}. \qquad (14)$$

We thus find $\Delta E^2 \ll 4kT^2C_{\mathrm{l}}$ if $G_{\mathrm{lb}} \ll G_{\mathrm{ml}}$, a condition established in the experiments. Note that the effective bandwidth of the integral in Eq. (12) is therefore limited by the phonon exchange between the lattice and the magnetic system. When the SQUID noise is included as well, the effective bandwidth may be further reduced.

In our experiments the noise of the SQUID-sensor dominates the thermodynamic noise of the detector. It is determined by the electronics at room temperature, and it therefore shows a $1/f$ component below 1 Hz. Besides that a white noise with a level of about $3 \times 10^{-6}\Phi_0/\sqrt{\mathrm{Hz}}$ has been found [4]. In our case the white noise corresponds to a resolution of absorbed energies of 8 $\mathrm{eV_{rms}}/\sqrt{\mathrm{Hz}}$ for a single pulse, or 40 $\mathrm{eV}/\sqrt{\mathrm{Hz}}$ for the FWHM of a pulse height spectrum, respectively [4]. Our detector has an absorber of 100 g of silicon at a temperature of 30 mK. If one plans to use a second SQUID as preamplifier, than the intrinsic noise of the SQUID-sensor would become relevant. In this case the sensor noise would be of the same order of magnitude as the term $4kT^2C_{\mathrm{l}}$ with the considered absorber of 100 g of silicon. Note that C_{l} is already small compared to C_{m}, which might have been expected to limit the sensitivity.

5. The magnetic sample

We have mentioned above that at typical temperatures of measurement (30 mK) the heat capacities are such that $C_{\mathrm{m}} \gg C_{\mathrm{l}}$. For calculation we assume therefore $C_{\mathrm{m}} = 100\ C_{\mathrm{l}}$. This is necessary because the signal rise time becomes short enough only under this condition for most of the samples investigated up to now [6]. The thermal conductance G_{lb} is established by point contacts such that the relaxation time $\tau_{\mathrm{b}} \cong C_{\mathrm{m}}/G_{\mathrm{lb}}$ is of the order of 100 s which can be measured directly as the signal decay time. With that we have a thermal relaxation between the lattice (C_{l}) and the bath of 1 s. This time is long enough in order to avoid that a considerable part of the absorbed energy leaks off from the lattice to the bath, before being transferred to the magnetic system. This transfer time $\tau_{\mathrm{s}} \cong C_{\mathrm{l}}/G_{\mathrm{ml}}$ is the signal rise time which is of the order of 10 ms or less.

Having these experimental conditions it is however laborious to cool down the detector to very low temperatures. This problem becomes even more serious because C_{m}, which is established by the Zeeman splitting of the magnetic atoms in the applied field, is not the only heat capacity of the magnetic atoms. In addition we have generally a heat capacity arising from interactions between the magnetic atoms, and that can be even much higher than C_{m} within a particular range of temperature. The best solution of the problem to cool down rapidly the detector and at the same time to establish a low conductance G_{lb} at the measuring temperature would be a movable mechanical contact. This technique has however not yet been used up to now.

Seidel et al. [7] have proposed very recently to use diluted magnetic samples which are metallic and exhibit therefore very short spin lattice relaxation times. In a first experiment they have actually found a signal rise time of 10 μs. In that case one need not establish $C_{\mathrm{m}} \gg C_{\mathrm{l}}$ for obtaining fast signals [8]. However, with metallic samples C_{l} includes the high contribution of the conduction electrons, of course. Furthermore, the spin–spin interaction via conduction electrons should be even stronger and that would strongly limit the allowed concentration of the magnetic atoms in the sample. Another possibility is to use insulators which exhibit short spin lattice relaxation times, e.g. $\mathrm{CaF_2:Ce}$. However, in this sample the short relaxation time is also due to an additional heat capacity, which we have proved at low temperatures, and which arises presumably from interstitial $\mathrm{F^-}$ ions. We assume that generally an additional heat capacity must be taken into account in order to reach short spin lattice relaxation times; the question is what will be the best approach. In conclusion we want to say that the ideal magnetic material for the magnetic bolometer is still a more or less open question.

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